12 August 1964

MEMORANDUM FOR RECORD

SUBJECT: The Change in Ground Resolution as a Function of Obliquity.

The equivalent question is: Given a certain angular resolution (ground target subtending an angle with a radius equal to the distance from target to lens), what is the effect of changing obliquity (aiming angle) on the size of a ground target just resolvable? Scale numbers will be used to show this functional relationship and are defined as the ratio of an object dimension--in a stated orientation--to the corresponding image dimension. A ground object "resolved" is arbitrarily defined herein to be a line and space pair just discernable or distinguishable. The following brief analysis shows that the orientation of a ground object is most important in determining whether it can be resolved, and this orientation is of great importance in determining system capability.

Assumptions and Comments

a. Only geometric effects are considered--atmospheric seeing is not included.

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b. A flat earth is assumed--using a spherical earth increases the discrepance in size between the resolvable dimentions of ground objects as a function of target orientation.

c. Camera dynamics (including image motion are considered to be 0).

d. The equations have been simplified to be applicable only at the principle point--except for $S_{\parallel}$ scales that are valid along a ground line parallel to the flight path.

<u>Scale Numbers for Ground Object's Oriented Normal to the Flight Path ($S_{\perp}$). See Figure 1.</u>

$$S_{\perp} = \frac{S}{I} = \frac{\frac{H}{\text{Cos } t}}{f} = \frac{H}{f} \text{ Sec } t \qquad (1)$$

$$\boxed{S_{\perp} = \frac{H}{f} \text{ Sec } t} \qquad (2)$$

<u>Scale Numbers for Ground Objects Oriented Parallel to the Flight Path ($S_{\parallel}$). See Figure 2.</u>

$$S_{\parallel} = \frac{Y(\text{actual object dimension})}{I(\text{actual image dimension})} \qquad (3)$$

We want to determine $S_{\parallel} = \frac{Y}{I}$, but there is no direct relationship apparent from the figure. There is, however, a direct relationship available bewteen S and I and between Y and S so we may determine the functional relationship between Y and I in terms of the parameter S.

$$\frac{Y}{S} \approx \text{Sec } t \text{ for small values of } Y \text{ and } S. \qquad (4)$$

$$\lim_{d\phi \to 0} \frac{\Delta Y}{\Delta S} = \text{Sec } t \qquad (5)$$

Now substituting from (1) and (5) and assuming Y and S to be small:

$$S_{\parallel} = \cancel{\frac{S}{I} \cdot \frac{Y}{S}} = \frac{H}{f} \text{Sec } t \cdot \text{Sec } t \qquad (6)$$

$$\boxed{S_{\parallel} = \frac{H}{f} \text{Sec}^2 t} \qquad (7)$$

<u>Scale Numbers for Ground Objects Oriented Vertically (S_h). See Figure 3.</u>

$$S_h = \frac{H(\text{actual object dimension})}{I(\text{actual image dimension})} \qquad (8)$$

We want to determine $S \;\; = \frac{H}{I}$, but there is no direct relationship apparent from the figure. There is, however, a direct relationship available between H and Y, and between Y and I, so we may determine the functional relationship between H and I in terms of the parameter Y.

$$\frac{H}{Y} \approx \text{Cot } t \text{ for small values of } H \text{ and } Y. \qquad (9)$$

$$\lim_{d\phi \to 0} \frac{\Delta H}{\Delta Y} = \text{Cot } t \qquad (10)$$

Substituting from (3), (7) and (10):

$$S_h = \cancel{\frac{H}{Y} \cdot \frac{Y}{I}} = \text{Cot } t \cdot \frac{H}{f} \text{Sec}^2 t \qquad (11)$$

Simplifying:

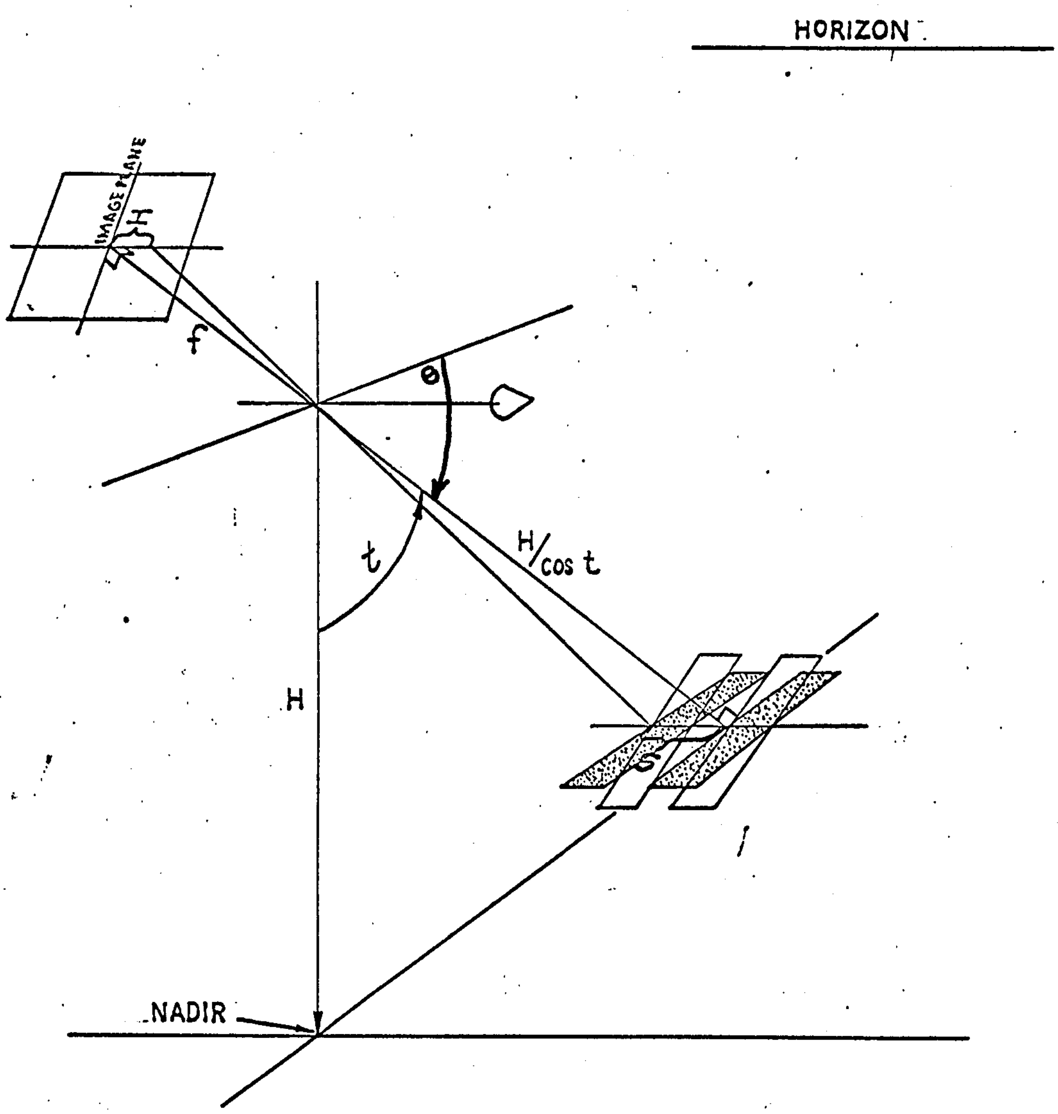
$$S_h = \frac{H}{I} = \frac{H}{f}\,\frac{\text{Cos } t}{\text{Sin } t} \cdot \frac{1}{\text{Cos}^2 t} = \frac{H}{f}\,\frac{1}{\text{Sin } t \text{ Cos } t} \qquad (12)$$

$$= \frac{H}{f}\,\frac{1}{\frac{1}{2}\text{ Sin } 2t} = \qquad (13)$$

$$\boxed{S_h = \frac{2H}{f}\text{ Csc } 2t} \qquad (14)$$

Figure 4 shows graphically the increase in size of ground objects resolved as the obliquity (Aiming Angle) increases due to the geometric projection involved--The deleterious effects of the atmosphere in the higher obliquity positions is not considered. These data obtained from the above derived equations are applicable for any camera type (frame, panoramic, or strip) for data near the principal point which is the intersection of the lens axis with the focal plane.

The graph is normalized for a ground resolution of one foot. These resolution numbers can be considered scalers for other ground resolutions. For example, if a certain camera system can just resolve a three foot object at the nadir, we can multiply all the ground resolution numbers on the chart by three.



$$S_{\perp} = \frac{S}{I}$$

Figure 1. $S_{\perp}$ Scale Numbers

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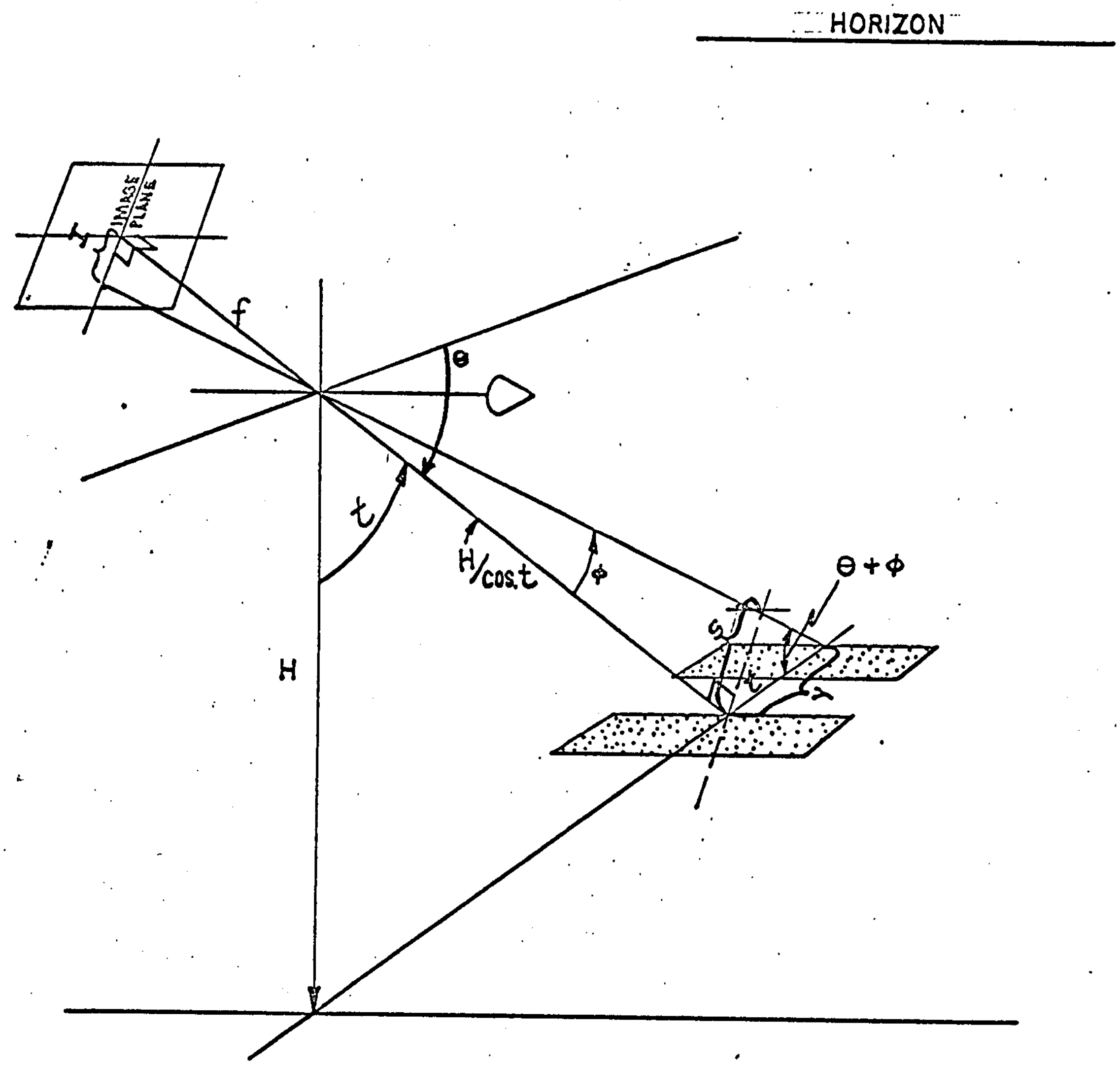
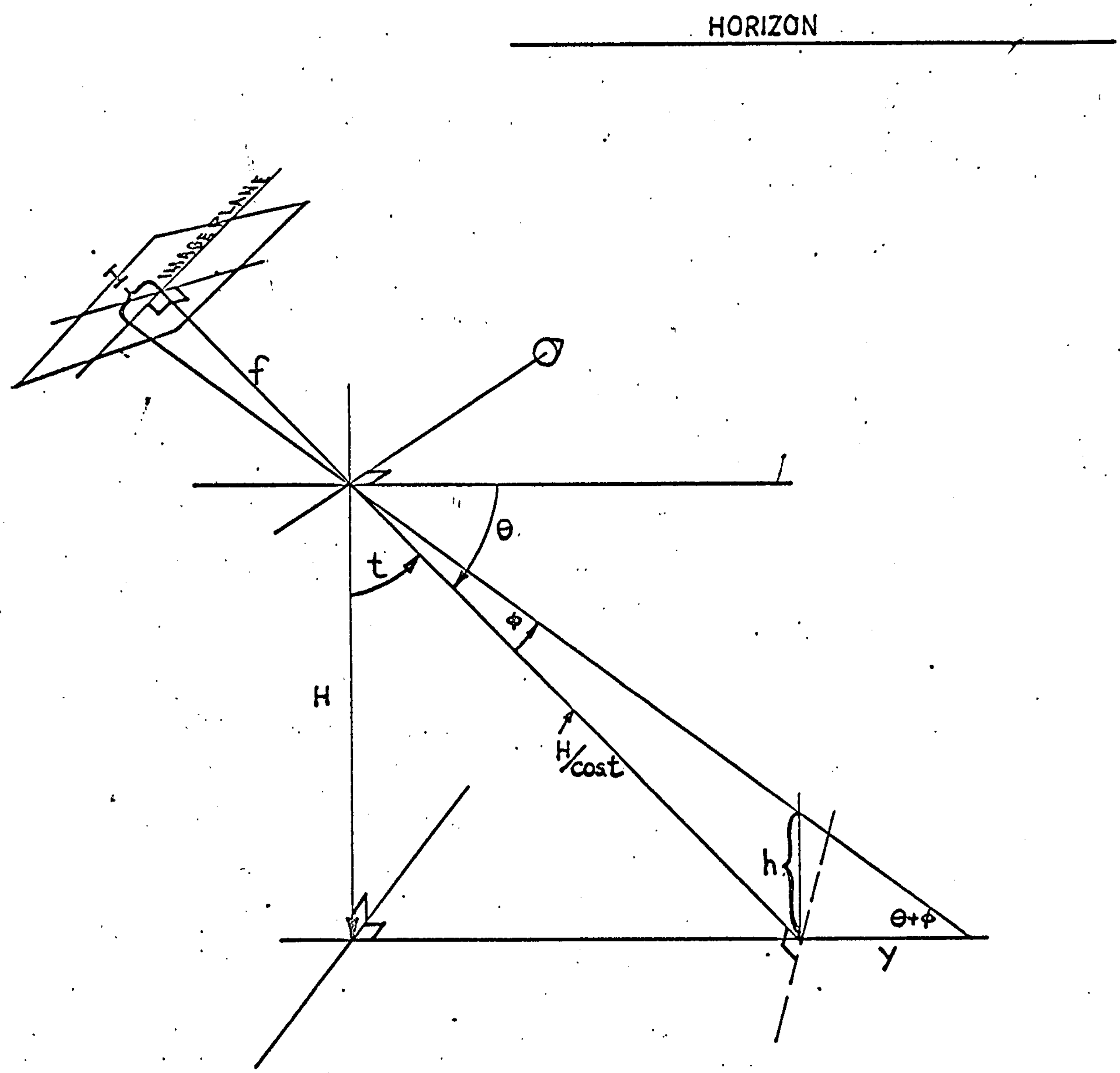


$$S_{11} = \frac{y}{I}$$

Figure 2. S_{11} Scale Numbers



$$S_h = \frac{h}{I}$$

Figure 3. S_h Scale Numbers

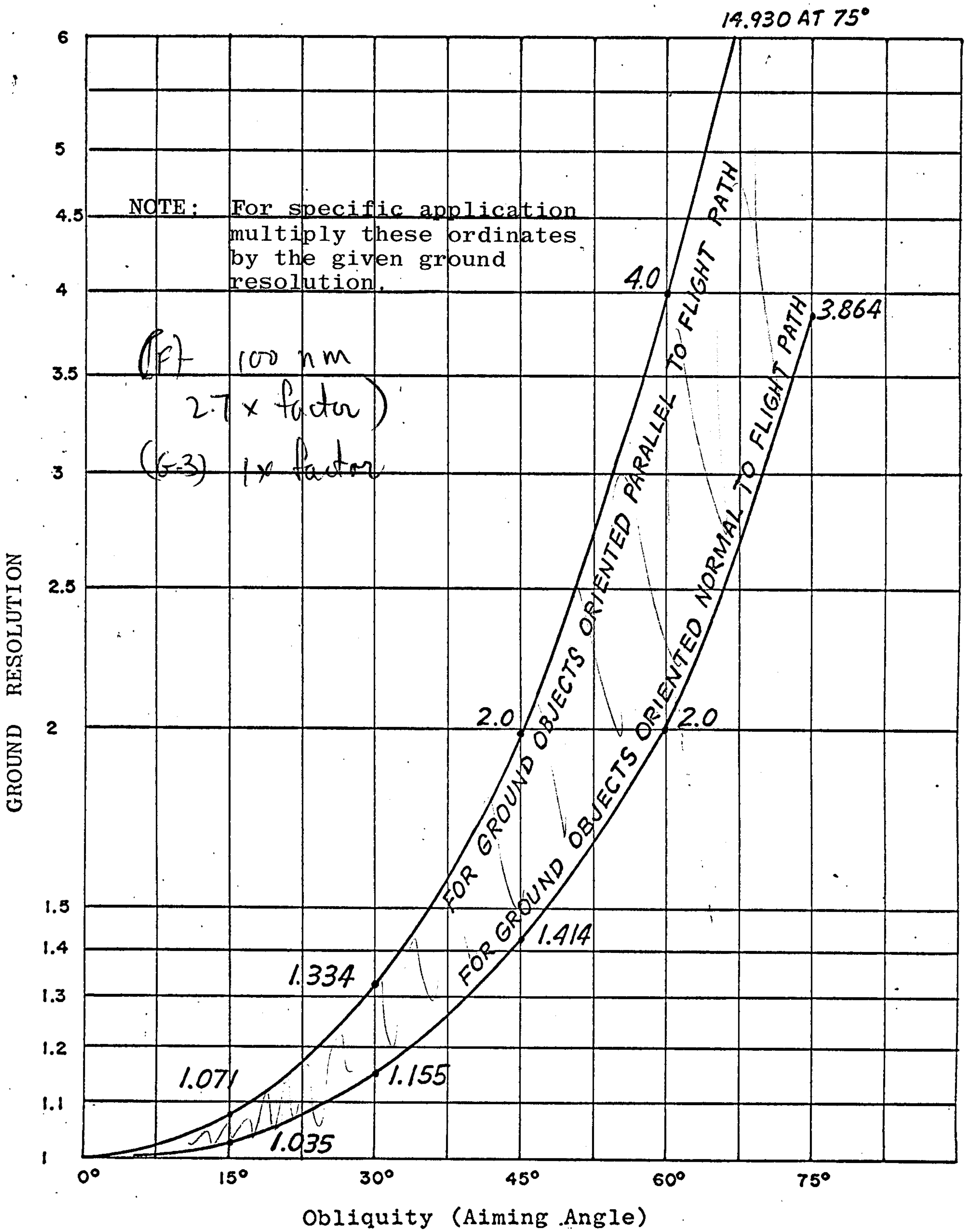

14.930 AT 75°
6
5
4.5
4
3.5
3
2.5
2
1.5
1.4
1.3
1.2
1.1
1
NOTE: For specific application multiply these ordinates by the given ground resolution.
(F) 100 nm
2.7 x factor)
(G-3) 1x factor
GROUND RESOLUTION
4.0
3.864
2.0
2.0
1.414
1.334
1.155
1.071
1.035
FOR GROUND OBJECTS ORIENTED PARALLEL TO FLIGHT PATH
FOR GROUND OBJECTS ORIENTED NORMAL TO FLIGHT PATH
0°
15°
30°
45°
60°
75°
Obliquity (Aiming Angle)

Figure 4.